ENGINEERING ECONOMICS REVIEW

For the

Louisiana Professional Engineering Examination

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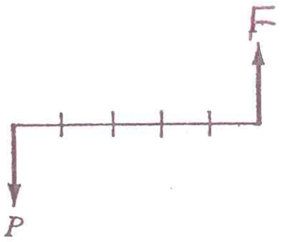
Contents



To Find F
Given P $(F/P,i\%,n)$

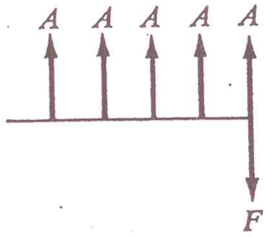
$$F = P(1 + i)^n$$

COMPOUND AMOUNT

To Find P
Given F $(P/F,i\%,n)$

$$P = F(1 + i)^{-n}$$

PRESENT WORTH



To Find A
Given F $(A/F,i\%,n)$

$$A = F\left[\frac{i}{(1 + i)^n - 1}\right]$$

SINKING FUND

To Find F
Given A $(F/A,i\%,n)$

$$F = A\left[\frac{(1 + i)^n - 1}{i}\right]$$

SERIES COMPOUND AMOUNT

i = interest rate per interest period. In the equations the interest rate is stated as a decimal (that is, 5% interest is 0.05).

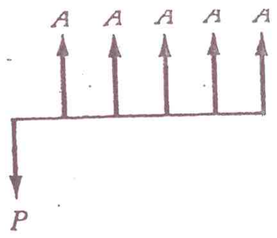
n = number of interest periods

P = a present sum of money

F = a future sum of money. The future sum F is an amount, n interest periods from the present, that is equivalent to P with interest rate i.

A = an end-of-period cash receipt or disbursement in a uniform series continuing for n periods, the entire series equivalent to P or F at interest rate i.

G = a uniform arithmetic gradient representing a period-by-period increase in payments or disbursements.



To Find A Given P $(A/P,i\%,n)$

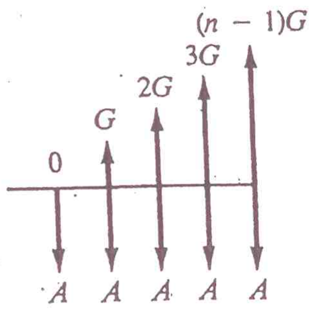
$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

CAPITAL RECOVERY

To Find P Given A $(P/A,i\%,n)$

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

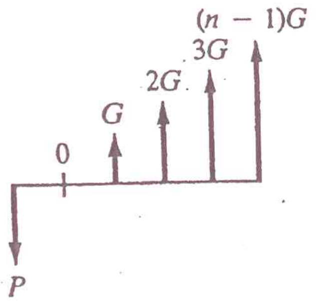
SERIES PRESENT WORTH



To Find A Given G $(A/G,i\%,n)$

$$A = G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

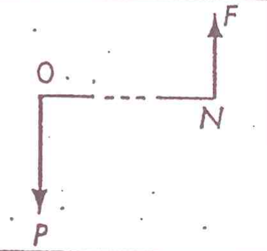
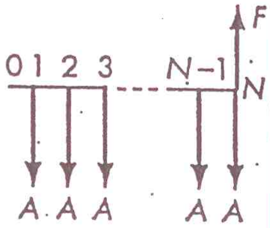
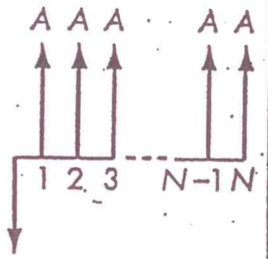
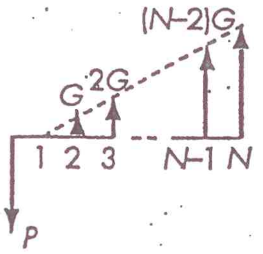
GRADIENT UNIFORM SERIES
(Arithmetic Gradient To Uniform Series)



To Find P Given G $(P/G,i\%,n)$

$$P = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i} - n\right]\left[\frac{1}{(1+i)^n}\right]$$

GRADIENT PRESENT WORTH
(Arithmetic Gradient To Present Worth)

Flow Type	Factor Notation	Formula	Cash Flow Diagram	Factor Relationship
SINGLE	Compound amount $(F/P, i, N)$	$F = P(1+i)^N$		$F = P(F/P, i, N)$
	Present worth $(P/F, i, N)$	$P = F(1+i)^{-N}$		$P = F(P/F, i, N)$
EQUAL PAYMENT SERIES	Compound amount $(F/A, i, N)$	$F = A\left[\frac{(1+i)^N - 1}{i}\right]$		$F = A(F/A, i, N)$
	Sinking fund $(A/F, i, N)$	$A = F\left[\frac{i}{(1+i)^N - 1}\right]$		$A = F(A/F, i, N)$
	Present worth $(P/A, i, N)$	$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$		$P = A(P/A, i, N)$
	Capital recovery $(A/P, i, N)$	$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$		$A = P(A/P, i, N)$ $A = iP \quad N \to \infty$
GRADIENT	Linear gradient Present worth $(P/G, i, N)$	$P = G\left[\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right]$		$P = G(P/G, i, N)$ $A = G(A/G, i, N)$ $F = P(F/P, i, N) = A(F/A, i, N)$ $F = G(F/G, i, N)$

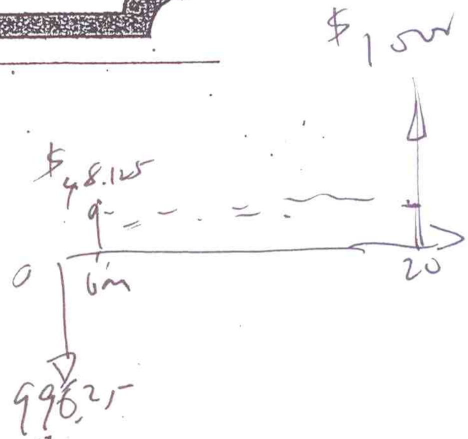
BONDS

Atlanta Gas & Light

New Issue/January 30, 1991
$85,000,000

Atlanta Gas & Light Company

First Mortgage Bonds, 9 5/8% Series Due 2001
Price 99.625% and accrued interest from February 1, 1991



Par Value: Stated face value $1,000.00

Maturity Date: January 31, 2001

Coupon Rate: 9 5/8 % 9.625 1 yr. 4.8125 % Semi-annual

Yield to Maturity: (YTM) Actual interest earned from a bond over the holding period

Discount or Premium Bond: Depends on the market price ($996.25)

Current Yield: Annual interest as a % of the current market price

48.13/996.25 = 4.83 % Semi-annually

Nominal = 9.66 %

Effective = 9.90 %

Effective Interest Rates

r % = Nominal Interest / year

m = Number if Interest Periods / year

r/m % = Interest Rate / sub period

Effective Annual Rate (i_a) = $(1 + r/m\ \%)^m - 1$

$$F = P(1 + (r/m)\%)^n = P(1 + (r/m))^{Nm}$$

n = Number of Interest Periods = Nm

N = Number of years

$$F = P(1 + i_a)^N$$

$$(1 + r\%/m)^m = [\,(1 + 1/(m/r))^{m/r}]^r$$

$m \rightarrow \infty \qquad (m/r) \rightarrow \infty$

$$\lim_{x \rightarrow \infty} (1+(1/x))^x = e = 2.718$$

$$m \rightarrow \infty \quad (1+1/(m/r))^{m/r} \rightarrow e$$

$F = Pe^r \qquad N = 1$

$F = Pe^{Nr}$ Continuous

$i_a = e^r - 1$ Continuous

INFLATION

PC: Present Cost of a commodity
FC: Future Cost of the commodity

$$FC = PC\,(1+\lambda)^n$$

λ : Inflation rate
n: Number of years

F_w: Future worth in today's dollars of a present amount P, held w/a i.

$$F_w = P \,/\, (1+\lambda)^n$$

If "i" is being compounded at the same time that inflation is occurring

$$F = P(1+i)^n \,/\, (1+\lambda)^n$$

$$= P(1+i)^n \,/\, (1+\lambda)^n = P\,[1 + (i-\lambda) \,/\, (1+\lambda)]^n$$

$$= P(1+\theta)^n = P(1+i^*)^n$$

$$i^* = \theta = (i-\lambda) \,/\, (1+\lambda)$$

Taxes: Rate "t"

$$I' = I - T = I - t\,i\,P = (1-t)\,i\,P$$

$$\theta = [((1-t)\,i - \lambda) \,/\, (1+\lambda)]$$

Adjusted-Discount Method

Effective interest rate:

$$(1+i') = (1+i_{,})(1+\lambda) = 1 + i_{,} + \lambda + i_{,}\lambda$$

$$d = i' = i_{,} + \lambda + i_{,}\lambda = i_{,} + f + i_{,} f$$

$i_{,}$ = 'Inflation - free' interest rate : True earning power

$\lambda = f$ = Inflation rate

$d = i'$ = 'Inflation - adjusted' MARR : Market Interest Rate

EXAMPLE:

What is the **present worth*** of \$5,000 in 5 years at $i_{,} = 10\%$ and inflation of 6%? * In constant (today's) dollar.

Market Rate d (i)

$$d = 0.1 + 0.06 + 0.1(0.06) = .166$$

$$P = 5000(1 + .166)^{-5} = \$\ 2,320.00$$

Net Present Value, Rate of Return, Payback Period, Benefit-Cost Ratio

This chapter continues the ideas developed in Chapter 7, particularly as they are applied in deciding among alternative capital investments.

8.1 NET PRESENT VALUE

The definition, (*7.1*), of the NPV is repeated here:

$$NPV = -|CF_0| + \sum_{j=1}^{n} CF_j\,(P/F, i\%, j) \tag{8.1}$$

in which the notation emphasizes our assumption that the initial cash flow, CF_0, is negative (a capital outlay). No assumption is made concerning the signs of the remaining CF_j, although often these terms will all be positive (revenues). In the special case $CF_j = A \quad (j = 1, 2, \ldots, n)$, (*8.1*) becomes, in view of (*3.4*),

$$NPV = -|CF_0| + A\,(P/A, i\%, n) \tag{8.2}$$

as it must. Another name for the NPV is the *discounted cash flow*, or DCF.

From (*8.1*) it is seen that the NPV is positive when and only when the total value of the returns CF_j (in year 0 dollars) exceeds the amount invested, $|CF_0|$ (year 0 dollars); that is to say, when and only when the original amount, earning compound interest at rate i for n years, would be insufficient to generate the returns. For a proposed investment to be economically acceptable, the NPV must be positive or, at worst, zero (in which case the investment of $|CF_0|$ would just suffice to yield the revenues CF_j).

Example 8.1 The cash flows associated with a milling machine are $CF_0 = -\$50\,000$, $CF_j = \$15\,000$ $(j = 1, \ldots, 5)$. Use (*8.2*) to determine the economic acceptability of this machine at interest rates of (*a*) 10%, (*b*) 15%, and (*c*) 20% per year, all compounded annually.

(*a*) $NPV = -\$50\,000 + \$15\,000(P/A, 10\%, 5) = -\$50\,000 + \$15\,000(0.26380)^{-1} = \$6861.26$

(*b*) $NPV = -\$50\,000 + \$15\,000(P/A, 15\%, 5) = -\$50\,000 + \$15\,000(0.29832)^{-1} = \$281.58$

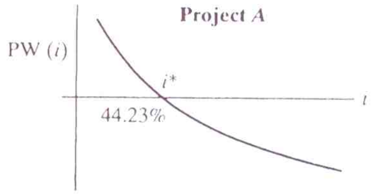
(*c*) $NPV = -\$50\,000 + \$15\,000(P/A, 20\%, 5) = -\$50\,000 + \$15\,000(0.33438)^{-1} = -\$5140.85$

The machine is seen to be an economically acceptable investment when the interest rate is 10%, and (barely) when the interest rate is 15%. It is *not* economically justifiable to buy the machine if the interest rate is 20%.

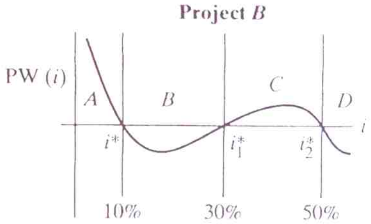
Rate of Return

Period (N)	Project A	Project B	Project C
0	−$1,000	−$1,000	$1,000
1	−$500	$3,900	−$450
2	$800	−$5,030	−$450
3	$1,500	$2,145	−$450
4	$2,000		

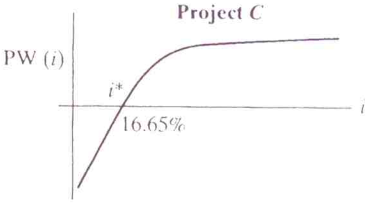
Project *A* is a simple investment.
Project *B* is a nonsimple investment.
Project *C* is a simple-borrowing cash flow.



(a)



(b)



(c)

Finding i^* by Direct Solution: Two Flows and Two Periods

Consider two investment projects with the following cash-flow transactions:

N	Project 1	Project 2
0	−$3,000	−$2,000
1	$0	$1,300
2	$0	$1,500
3	$0	
4	$4,500	

Compute the rate of return for each project.

Given: Cash flows for two projects.
Find: i^* for each project.

SOLUTION

Project 1: Solving for i^* in $PW(i^*) = 0$ is identical to solving for i^* in $FW(i^*) = 0$, because FW equals PW times a constant. We could use either method here, but we choose $FW(i^*) = 0$. Using the single-payment future-worth relationship, we obtain

$$FW(i) = -\$3{,}000(F/P, i, 4) + \$4{,}500 = 0.$$

Setting $FW(i) = 0$, we obtain

$$\$4{,}500 = \$3{,}000(F/P, i, 4) = \$3{,}000(1 + i)^4,$$

or

$$1.5 = (1 + i)^4.$$

Solving for i yields

$$i^* = \sqrt[4]{1.5} - 1 = 0.1067, \text{ or } 10.67\%.$$

Project 2: We may write the PW expression for this project as follow

$$PW(i) = -\$2{,}000 + \frac{\$1{,}300}{(1 + i)} + \frac{\$1{,}500}{(1 + i)^2} = 0.$$

Let

$$X = \frac{1}{(1 + i)}.$$

Given $aX^2 + bX + c = 0$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE 7.3 Finding i by Trial and Error

ACME Corporation distributes agricultural equipment. The board of directors is considering a proposal to establish a facility to manufacture an electronically controlled "intelligent" crop sprayer invented by a professor at a local university. This crop-sprayer project would require an investment of \$10 million in assets and would produce an annual after-tax net benefit of \$1.8 million over a service life of eight years. All costs and benefits are included in these figures. When the project terminates, the net proceeds from the sale of the assets would be \$1 million (Figure 7.2). Compute the rate of return of this project.

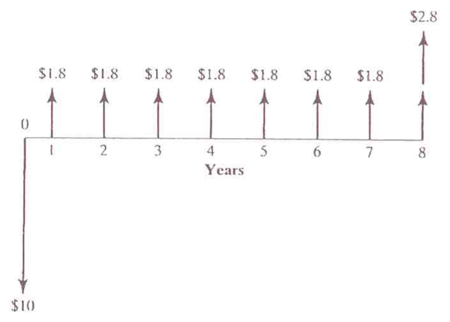


Figure 7.2 Cash flow diagram for a simple investment
All dollar amounts are in millions of dollars

DISSECTING THE PROBLEM

Given: I = \$10 million, A = \$1.8 million, S = \$1 million, and N = 8 years.

Find: i^*.

METHODOLOGY

Find i^* by trial and error.

SOLUTION

We start with a guessed interest rate of 8%. The net present worth of the cash flows in millions of dollars is

$$PW(8\%) = -\$10 + \$1.8(P/A, 8\%, 8) + \$1(P/F, 8\%, 8) = \$0.88.$$

Since this net present worth is positive, we must raise the interest rate in order to bring this value toward zero. When we use an interest rate of 12%, we find that

$$PW(12\%) = -\$10 + \$1.8(P/A, 12\%, 8) + \$1(P/F, 12\%, 8) = -\$0.65.$$

We have bracketed the solution. PW(i) will be zero at i somewhere between 8% and 12%. Using straight-line interpolation, we approximate that

$$\begin{aligned} i^* &\approx 8\% + (12\% - 8\%)\left[\frac{0.88 - 0}{0.88 - (-0.65)}\right] \\ &= 8\% + 4\%(0.5752) \\ &= 10.30\%. \end{aligned}$$

Now we will check to see how close this value is to the precise value of i^*. If we compute the net present worth at this interpolated value, we obtain

$$\begin{aligned} PW(10.30\%) &= -\$10 + \$1.8(P/A, 10.30\%, 8) + \$1(P/F, 10.30\%, 8) \\ &= -\$0.045. \end{aligned}$$

As this result is not zero, we may recompute i^* at a lower interest rate, say, 10%:

$$\begin{aligned} PW(10\%) &= -\$10 + \$1.8(P/A, 10\%, 8) + \$1(P/F, 10\%, 8) \\ &= \$0.069. \end{aligned}$$

With another round of linear interpolation, we approximate that

$$\begin{aligned} i^* &\approx 10\% + (10.30\% - 10\%)\left[\frac{0.069 - 0}{0.069 - (-0.045)}\right] \\ &= 10\% + 0.30\%(0.6053) \\ &= 10.18\%. \end{aligned}$$

At this interest rate,

$$\begin{aligned} PW(10.18\%) &= -\$10 + \$1.8(P/A, 10.18\%, 8) + \$1(P/F, 10.18\%, 8) \\ &= \$0.0007, \end{aligned}$$

which is practically zero, so we may stop here. In fact, there is no need to be more precise about these interpolations, because the final result can be no more accurate than the basic data, which ordinarily are only rough estimates. Incidentally, computing the i^* for this problem on a computer gives us 10.1819%.

Payback Period vs. Discounted Payback Period

Given: Initial cost = \$1,800,000; cash flow series as shown in the table below.

Find: Conventional-payback period.

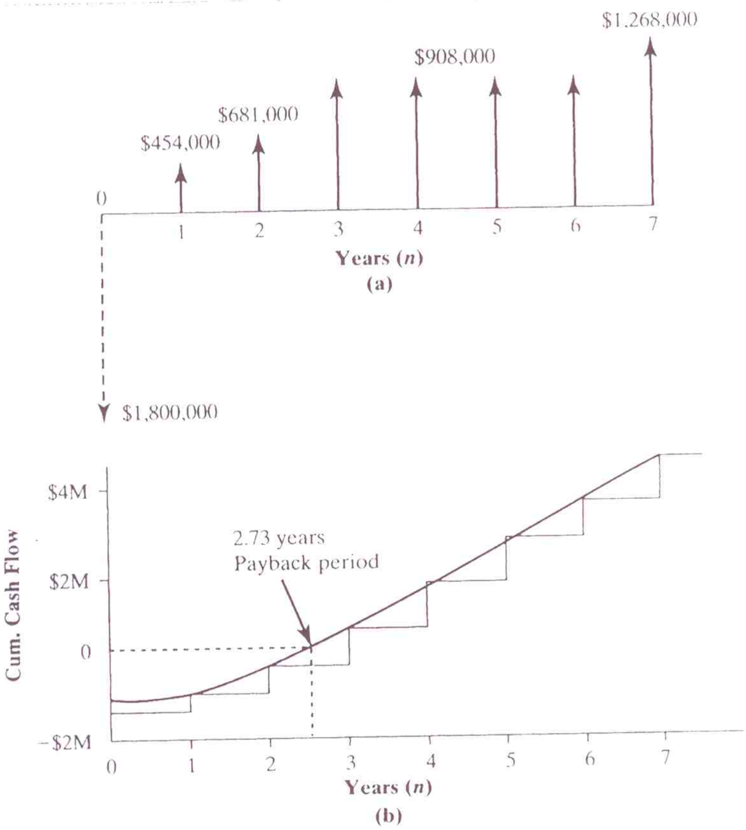
Period	Cash Flow	Cumulative Cash Flow
0	−\$1,800,000	−\$1,800,000
1	\$454,000	−1,346,000
2	681,000	−665,000
3	908,000	243,000
4	908,000	1,151,000
5	908,000	2,059,000
6	908,000	2,967,000
7	1,268,000	4,235,000

Payback Period Calculation Considering the Cost of Funds at 15%

Period	[illegible]	[illegible]	[illegible]
0	−\$1,800,000	0	−\$1,800,000
1	454,000	−1,800,000(0.15) = −270,000	−1,616,000
2	681,000	−1,616,000(0.15) = −242,400	−1,177,400
3	908,000	−1,177,400(0.15) = −176,610	−446,010
4	908,000	−446,010(0.15) = −66,902	395,089
5	908,000	395,089(0.15) = 59,263	1,362,352
6	908,000	1,362,352(0.15) = 204,353	2,474,705
7	1,268,000	2,474,705(0.15) = 371,206	4,113,911

SOLUTION

See Figure 5.2.



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BENEFIT – COST ANALYSIS

Users' Benefits (B) = Benefits – Disbenefits

Primary Benefits / Secondary Benefits

Sponsor's Cost = Capital Costs (I)

\+ Operating and Maintenance Cost (C′)

\- Revenues

Interest Rate = Social Discount Rate

NPW (user's net benefit) > Sponsor's Net Costs

Benefit / Cost Ratios

$$B/C = B / (I + C')$$

I : Capital Expenditure

C′ : Operating Costs

Net B/C ratio, B′/C

$$B'C = (B\text{-}C') / I = B' / I$$

B′ : Net Benefit

Acceptance Rule:

$$B / (I+C') > 1 \quad , \quad (B\text{-}C') / I > 1$$

$$NPW(i) = B - C > 0$$

Example 14.1 Benefit-cost ratio

A public project being considered by a local government has the following estimated benefit-cost profile (see Fig. 14.1).

n	b_n	c_n	A_n
0		\$10	−\$10
1		10	−10
2	\$20	5	15
3	30	5	25
4	30	8	22
5	20	8	12

Assume that $i = 10\%$, $N = 5$, and $K = 1$. Compute B, C, I, C', $BC(10\%)$ and $B'C(10\%)$.

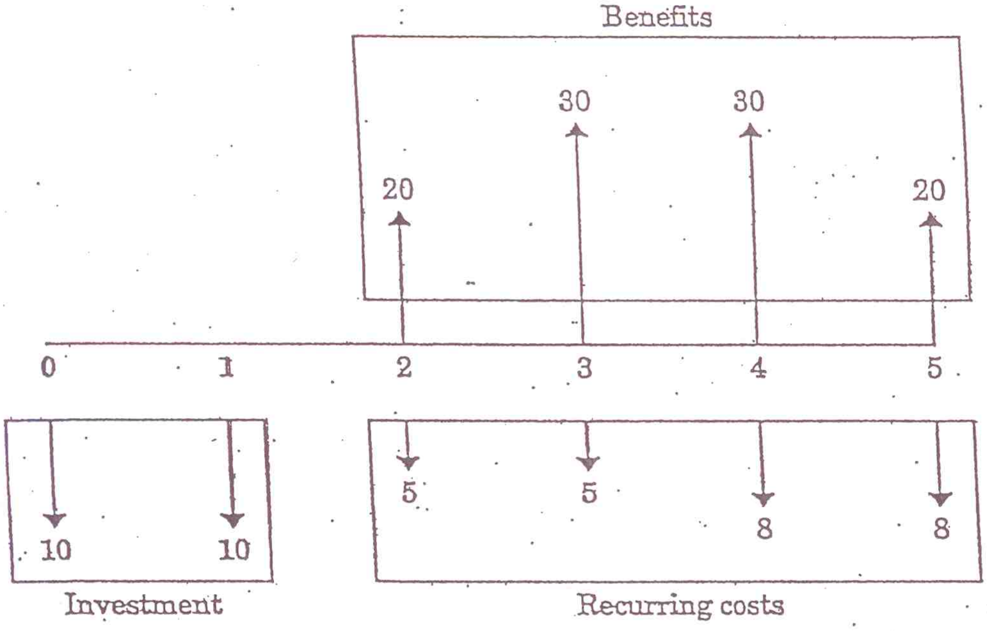


Figure 14.1 ■ Classification of project's cash flow elements

Solution

$$B = \$20(P/F, 10\%, 2) + \$30(P/F, 10\%, 3)$$
$$+ \$30(P/F, 10\%, 4) + \$20(P/F, 10\%, 5)$$
$$= \$71.98$$

$$C = \$10 + \$10(P/F, 10\%, 1) + \$5(P/F, 10\%, 2) + \$5(P/F, 10\%, 3)$$
$$+ \$8(P/F, 10\%, 4) + \$8(P/F, 10\%, 5)$$
$$= \$37.41$$

$$
\begin{aligned}
I &= \$10 + \$10(P/F, 10\%, 1) \\
&= \$19.09 \\
C' &= C - I \\
&= \$18.32
\end{aligned}
$$

Using Eqs. (14.5) and (14.6), we can compute the B/C ratios as

$$BC(10\%) = \frac{\$71.98}{\$19.09 + \$18.32} = 1.92 > 1$$

$$B'C(10\%) = \frac{\$71.98 - \$18.32}{\$19.09} = 2.81 > 1$$

The B/C ratios exceed 1, so the users' benefit exceeds the sponsor's cost.

Mutually Exclusive Public Projects:

Arrange Projects in an Increasing order of

$$(I + C')$$

$J \quad K \qquad \Delta B = B_K - B_J$

$$BC_{(K-J)} = \Delta B / (\Delta I + \Delta C') \qquad (1)$$

OR

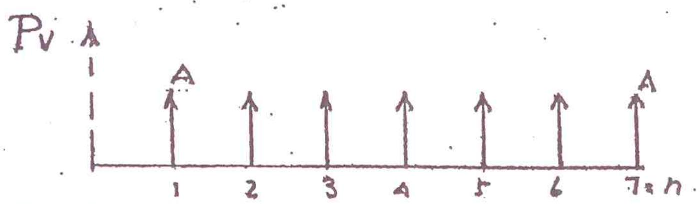
$$B'C_{(K-J)} = (\Delta B - \Delta C') / \Delta I \qquad (2)$$

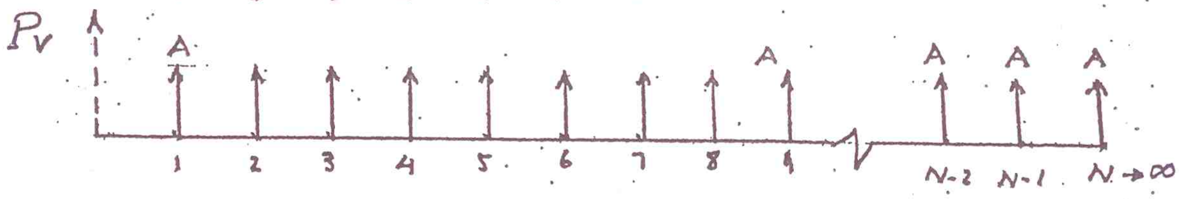
If $BC_{K-J} > 1$ then select K else select J

If $B_K = B_J$ equation (1) $\Delta B = 0$, $BC_{(K-J)} = 0$ uses equation (2)

$$B'C_{(K-J)} = (0 - \Delta C') / \Delta I$$

Capitalized Cost / Return





$P_v = A\,(P/A, i, n)$

$P_v = \dfrac{A}{i} \qquad N \to \infty \quad$ *long life / perpetual*

16

DEPRECIATION

Asset Depreciation

-Physical

-Functional

Economic Depreciation = Purchase Price – Market Value

Accounting Depreciation

-Book Depreciation

-Tax Depreciation

Net Income = After - Tax Profit

Net Income = Taxable Income – Income Taxes

Taxable Income = Gross Income (Revenues)
-Cost of Goods Sold
-Depreciation
-Operating Expenses

Income Taxes = Tax rate x Taxable Income

Factors of Asset Depreciation:

Depreciable Asset

Cost Base (I)

Salvage Value (S)

Depreciable Life

Book Value(n)= $I - \sum_{1}^{n} D_i$

Straight Line Method (SL)

$$D_n = \frac{(I - S)}{N}, \tag{7.1}$$

where

D_n = the depreciation charge during year n

I = the cost of the asset, including installation expenses

S = the salvage value at the end of useful life

N = the useful life

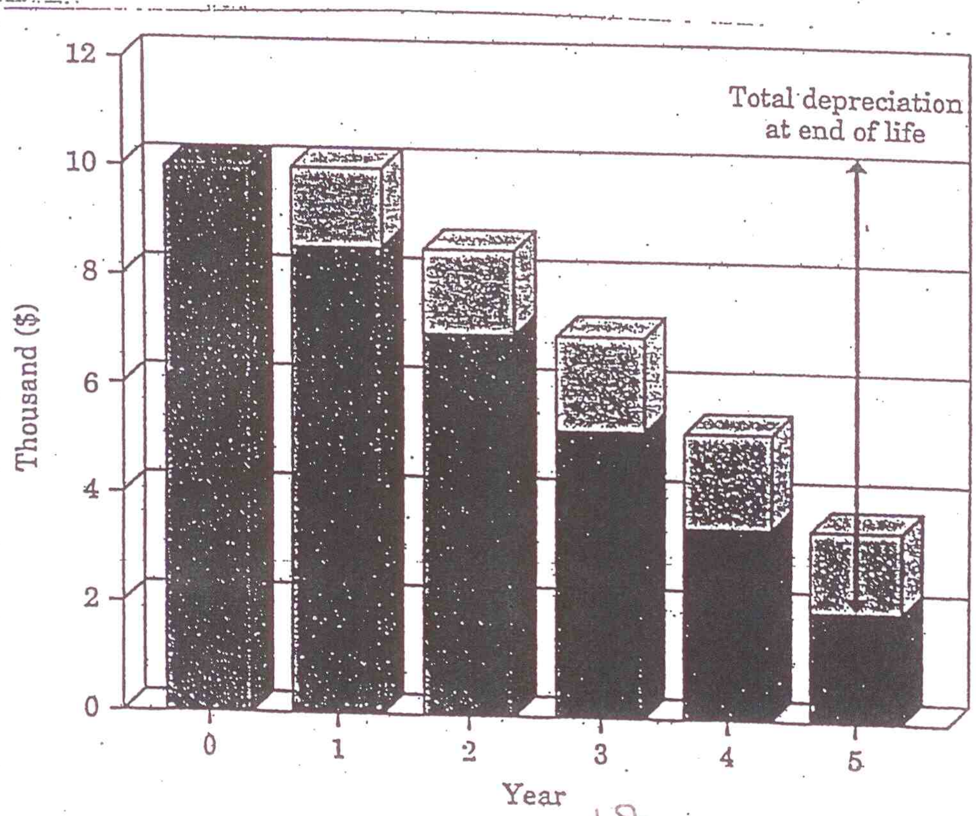
$1/N$ = the straight-line depreciation rate.

The book value of the asset at the end of n years is then defined as

book value = cost base − total depreciation charges made,

or

$$B_n = I - (D_1 + D_2 + D_3 + \cdots + D_n). \tag{7.2}$$



Accelerated Methods:

I- Declining Balance Method:

Allocates for depreciation a fixed fraction of the Initial Book Balance each year.

$$\alpha = \frac{1}{N} \text{ (Multiplier)}$$

Multipliers		
	1.5	(150 % DB)
	2.0	(200 % Double Declining Balance)

Total Depreciation

$$TDB = D_1 + D_2 + \ldots\ldots + D_n = \Sigma D_i$$

$$= I\,[1 - (1-\alpha)^n]$$

Book Value Bn

$$B_n = I - \sum_{i}^{n} D_i$$

$$= I\,(1-\alpha)^n$$

i	D_i	B_i
0		I
1	$I\alpha$	$I(1-\alpha)$
2	$I(1-\alpha)\alpha$	$I(1-\alpha)^2$
3	⋮	⋮
n	$I(1-\alpha)^{n-1}\alpha$	$I(1-\alpha)^n$

TABLE 8.8 MACRS Depreciation Schedules for Personal Property with Half-Year Convention

Year	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	33.33	20.00	14.29	10.00	5.00	3.750
2	44.45	32.00	24.49	18.00	9.50	7.219
3	14.81*	19.20	17.49	14.40	8.55	6.677
4	7.41	11.52*	12.49	11.52	7.70	6.177
5		11.52	8.93*	9.22	6.93	5.713
6		5.76	8.92	7.37	6.23	5.285
7			8.93	6.55*	5.90*	4.888
8			4.46	6.55	5.90	4.522
9				6.56	5.91	4.462*
10				6.55	5.90	4.461
11				3.28	5.91	4.462
12					5.90	4.461
13					5.91	4.462
14					5.90	4.461
15					5.91	4.462
16					2.95	4.461
17						4.462
18						4.461
19						4.462
20						4.461
21						2.231

- * Year to switch from declining balance to straight line. Source: IRS Publication 534, *Depreciation*, U.S. Governme[nt] Printing Offices, Washington, DC, December 1995.
- For illustration, MACRS percentages of 3-year property, beginning with the first tax year and ending with the fourth year, are computed as follows: Straight-line rate = 1/3, 200% DB rate = $2\left(\frac{1}{3}\right) = 0.6667$, $S = 0$.

Year	Calculation	MACRS percentage
1	$\frac{1}{2}$ year DDB dep = 0.5(0.6667)	= 33.33%
2	DDB dep = 0.6667(1 − 0.3333)	= 44.45%
	SL dep = $(1/2.5)(1 - 0.3333)$	= 26.67%
3	DDB dep = 0.6667(1 − 0.7778)	= 14.81%

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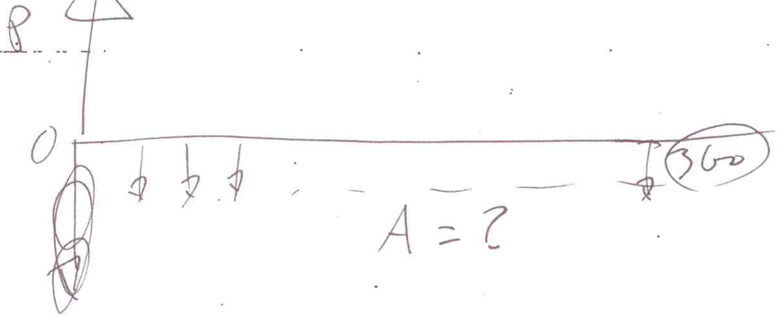
Examples

1-1. A \$105,000 mortgage loan is to be repaid in 360 monthly instalments at 9% nominal interest. The amount of each instalment is most nearly:

(A) \$790
(B) \$840
(C) \$845
(D) \$880

1-1 Solution. $r = 0.09$, $m = 12$, so $i = 0.09/12 = 0.0075$. $(P/A\ i, 360) = 124.2818656$. Given $P = 105{,}000$, the result is $A = P/(P/A\ i, 360) = \$844.85$, which is near \$845. Response C is correct.



1-2. To help pay for a new pipeline extension, a company borrows \$14,250 at 13% annual compound interest, to be repaid in two equal annual instalments. Interest on the loan is deductible. The amount of interest paid in the second year is most nearly:

nominal

$i = \frac{r}{1}$

(A) \$870
(B) \$980
(C) \$1850
(D) \$6690

1-2 Solution. The amount of the instalments is $A = 14{,}250/(P/A\ 13\%, 2) = 14{,}250/1.668102435 = \$8542.64$. The interest for the first year is $0.13 \times 14{,}250 = \$1852.50$, so the balance is reduced by the remainder $8542.64 - 1852.50 = \$6690.14$. The new balance is $14{,}250 - 6690.14 = \$7559.86$. The interest for the second year is $0.13 \times 7759.86 = \$982.78$. Response B is correct.

i	Interest	Principle	Balance
0			\$14250
1	\$1852.5	\$6690.14	7559.86
2			

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1-3. To replace a machine that has worn out, you can pay \$5000 for a machine that has a 3-year depreciation recovery period. An alternative to provide the same benefits is to repair the existing machine. If you use a 15% interest rate in after-tax analysis, and pay 40% income taxes, the maximum amount that you could afford to pay for the repair is most nearly:

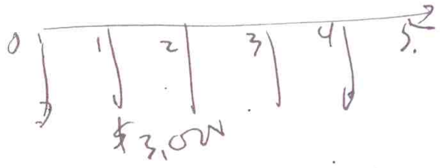
(A) \$1500
(B) \$4800
(C) \$5300
(D) \$5800

1-3 Solution. If repair is not chosen, the depreciation charges in years {1, 2, 3, 4} will be \$5000×{0.333, 0.445, 0.148, 0.074}, and the corresponding tax relief amounts will be T times these amounts, where $T = 0.40$; the resulting tax relief amounts are {\$666, \$890, \$296, \$148}. With $\beta = 1/1.15$, the present worth of the tax relief will be $666\beta + 890\beta^2 + 296\beta^3 + 148\beta^4 =$ \$1531.34.

On the other hand, if repair at cost X were chosen, the cost would be expensed, so that the tax relief in the first year would be $0.40X$ dollars, and the present worth of the tax relief would be $0.40X\beta = 0.347826087X$ dollars. You can afford to pay X dollars for repair if the present worth of the net after-tax cost (actual cost less tax relief) for repair does not exceed that for replacement: $X - 0.347826087X \leq 5000 - 1531.34 \rightarrow X \leq \$5318.61$. Response C is correct.

Note: In this problem, the tax rate actually is irrelevant, as can be seen by carrying T as a symbol and letting it cancel out of the inequality.

1-4. Annual 4% inflation is expected. A remote control panel can be leased for five years at a fixed cost of \$3000 per year, paid at the beginning of each year. It will save 125 labor hours per year. A labor hour is currently valued at \$22.50. At 14% interest before inflation, the present worth of this opportunity is most nearly:



(A) −\$640
(B) −\$580
(C) −\$180
(D) \$390

1-4 Solution. Given $i = 14\%$ and $f = 4\%$, we have $d = 0.14 + 0.04 + 0.14 \times 0.04 = 18.56\%$ for discounting after-inflation cash flows. The annual labor savings amount is $125 \times 22.50 =$ \$2812.50 before inflation. The present worth of the opportunity is $P = -3000(P/A, d, 5) + 2812.50(P/A, i, 5) = -3000(3.087913881) + 2812.50(3.433080968) = \$391.80$. Response D is correct.

1-5. A yogurt-making process ends with filling and packaging. You can purchase a new filler machine that can fill up to 700 liters per hour. Under this option (option A), fixed costs for filling and packaging will be \$89 per hour and variable costs will be \$0.067 per liter. Alternatively, you can purchase a combination machine that integrates filling with packaging and can handle up to 500 liters per hour. Under this option (option B), fixed costs will be \$105 per hour and variable costs will be \$0.024 per liter. A third alternative (option C) is to contract out the filling and packaging at \$0.50 per liter. Production will not exceed demand. The demand ranges (liters per hour) for which various options are best are most nearly:

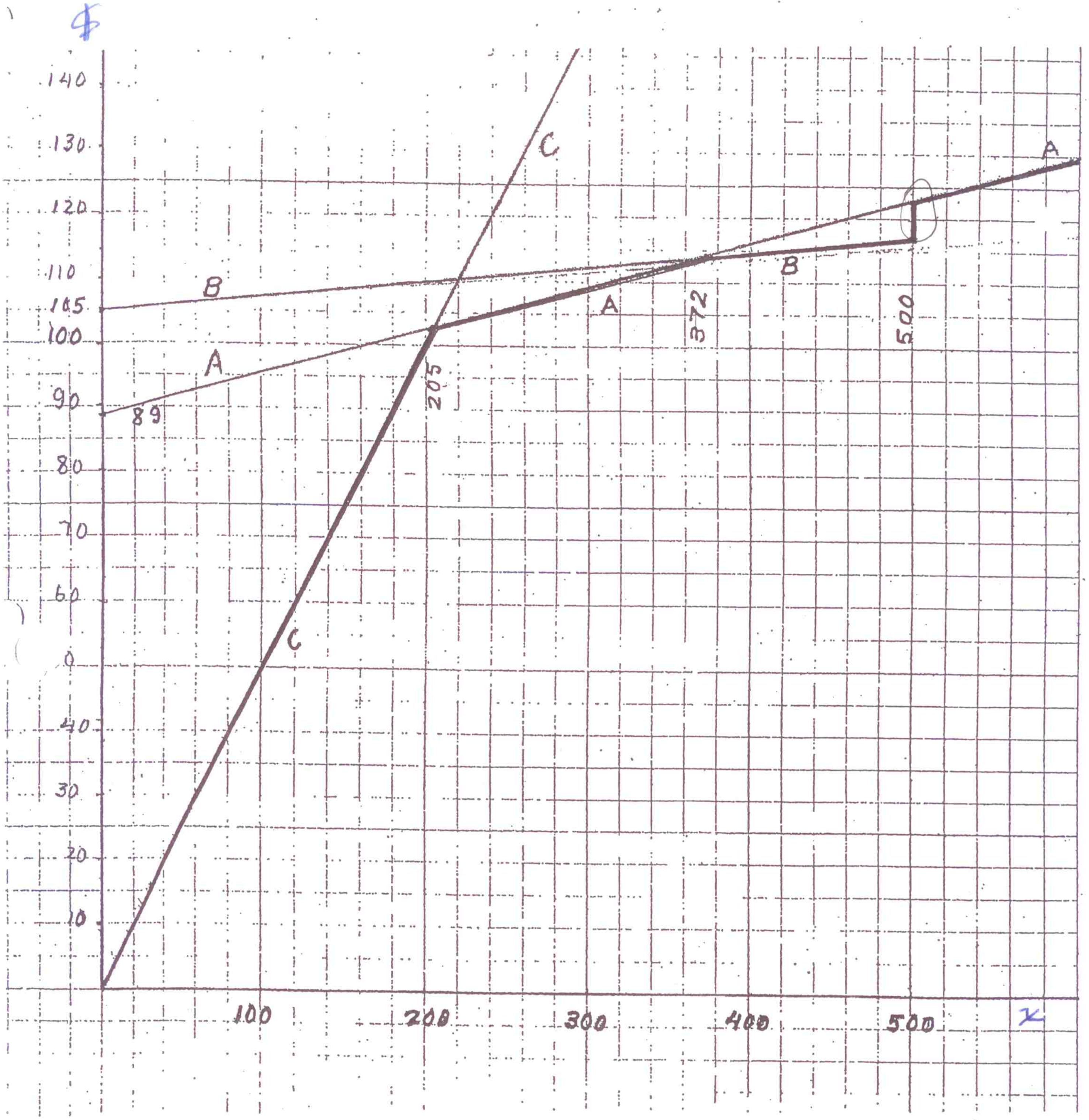
(A) C below 200, B between 400 and 500, otherwise A

(B) B below 400, A above 400

(C) C below 200, A above 200

(D) A below 400 and above 500, otherwise B

1-5 Solution. Let x be the production rate. In dollars per hour, the total of fixed and variable costs for the various options are $C^{(A)} = 89 + 0.067x$, $C^{(B)} = 105 + 0.024x$, and $C^{(C)} = 0.50x$. Obviously C is cheapest for small x. For C and A to have the same total costs, we have $0.50x = 89 + 0.067x \rightarrow x = 205.54$, and for C and A to have the same total costs, we have $0.50x = 105$ $0.024x \rightarrow x = 220.59$. In the neighborhood of these x values, A is cheaper than B. For A and B to have the same total costs, we have $89 + 0.067x = 105 + 0.024x \rightarrow x = 372.09$. For greater x, B is cheaper, but B can handle only up to $x = 500$, so A is the only option above 500. Response A is correct. (If the demand exceeded 700, you would produce 700 under option A, and the excess demand would presumably be filled under option C, but this is irrelevant to choosing a response since none of the listed responses mention C for high demand.) *Note*: Sketching graphs of total cost versus demand would probably be the easiest approach to solving this problem.


$
140
130
120
110
105
100
90
89
80
70
60
40
30
20
10
100
200
300
400
500
x
A
B
C
205
372
500

Engineering Economics

1. Kelly just won the state lottery. The $2,000,000 jackpot will be paid in 20 annual installments of $100,000. The first installment is given to Kelly immediately. The interest rate is 6% compounded yearly. The present worth of Kelly's lottery winnings (at the time of receiving the first installment) is most nearly
   (A) $1,115,810 (B) $1,146,990 (C) $1,215,810 (D) $1,900,000
2. Joe wants to be millionaire. To achieve this goal, he invests $5,000 each year into an account that pays 10% interest, compounded yearly. The amount of time it will take Joe to reach his goal of becoming a millionaire is
   (A) 20 yr (B) 32 yr (C) 43 yr (D) 181 yr
3. A credit card company offers students a credit line of $2,000 and charges an annual percentage rate of 12%, compounded daily. What is the effective annual interest rate?
   (A) 3.28% (B) 12.00% (C) 12.75% (D) 13.19%
4. Jane is planning for her retirement. Each month she places $200 in an account that pays 12% nominal interest, compounded monthly. She makes the first deposit of $200 on January 31, 1997. The last $200 deposit will be made on December 31, 2016. If the interest rate remains constant and all deposits are made as planned, the amount in Jane's retirement account on January 1, 2017 is most nearly
   (A) $60,000 (B) $155,000 (C) $173,000 (D) $198,000
5. Dawn purchased a $10,000 car. She put $2,000 down and financed the $8,000 balance. The interest rate is 9% nominal, compounded monthly, and the loan is to be repaid in equal monthly installments over the next four years. Dawn's monthly car payment is most nearly
   (A) $167 (B) $172 (C) $188 (D) $200
6. James is a major prizewinner in a sweepstakes. He has the option of either receiving a single check for $125,000 now or receiving a check for $50,000 each year for three years. (James will be given the first $50,000 check now.) At what interest rate would James have to invest his winnings for him to be indifferent as to how he receives his winnings?
   (A) 15.5% (B) 20.0% (C) 21.6% (D) 23.3%
7. A $105,000 mortgage loan is to be repaid in 360 monthly installments at 9% nominal interest. The amount of each installment is most nearly
   (A) $790 (B) $840 (C) $845 (D) $880
8. The help pay for a new pipeline extension, a company borrows $14,250 at 12% annual compound interest, to be repaid in two equal annual installments. Interest on the loan is deductible. The amount of interest paid in the second year is most nearly
   (A) $870 (B) $900 (C) $1850 (D) $6690
9. Annual 4% inflation is expected. A remote control panel can be leased for five years at a fixed cost of $3,000 per year, paid at the beginning of each year. It will save 125 labor hours per year. A labor hour is currently valued at $22.50. At 14% interest before inflation, the present worth of this opportunity is most nearly
   (A) -$640 (B) -$580 (C) -$180 (D) $390
10. Assume you have two alternatives for a worn out machine. You can replace the machine by paying $5,000 for a machine that has a 3-year MACRS depreciation recovery period or you can repair the existing machine. The maximum amount that you could afford to pay for the repair if 15% interest rate is used in after-tax analysis, and income tax rate is 40% is most nearly
    (A) $1,500 (B) $4,800 (C) $5,300 (D) $5,800

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